

## On the Mechanisms for Drag Reduction: A Viscoelastic Evaluation Based on a Bead-Spring Model

R. Y. TING, *Surface Chemistry Branch, Naval Research Laboratory,  
Washington, D. C. 20390*

### Synopsis

Two possible drag reduction mechanisms were examined by studying the viscoelastic effects of polymer solutions for the separate cases of oscillatory shear flow and elongational flow. The constitutive equation used was based on a modified dumbbell molecular model which predicts non-Newtonian viscosity and both the primary and the secondary normal stress differences. It can be shown that when this constitutive equation is arranged in the form of the Oldroyd model, the latter becomes a special case of this more general equation. The present results show that viscoelastic effects on the mean local rate of energy dissipation of a fluid element in an oscillatory motion are negligibly small. However, such effects introduce very large increases in the elongational viscosity as the stretching rate exceeds a certain limiting value and the flow time exceeds the terminal relaxation time of the fluid. The relative merits of these findings as possible explanations of turbulent drag reduction are briefly discussed.

### INTRODUCTION

It is well known that the addition of very small amounts of high molecular weight linear polymers to solvents in turbulent pipe flow can substantially reduce the pressure drop at a given flow rate. Toms<sup>1</sup> first observed this phenomenon in turbulently flowing solutions of poly (methyl methacrylate) in monochlorobenzene, and since that time many investigators have confirmed his findings. It can readily be demonstrated that a few parts per million of polyethylene oxide in water will lead to a reduction of the pure water friction in turbulent pipe flow by nearly 70% depending on the Reynolds number. However, in spite of the considerable interest and accomplishments in recent years, the basic mechanisms responsible for drag reduction are still unresolved.

The early work of Metzner and Park<sup>2</sup> suggested that viscoelastic effects might possibly be involved in the drag reduction phenomenon. These investigators carried out viscometric and drag reduction measurements in concentrated solutions and were able to correlate the reduction in wall shear stress with the first normal stress difference. In general, all polymers which exhibit drag reduction effects in dilute solution also show various viscoelastic effects at higher concentrations. Currently, two viscoelastic mechanisms have been proposed in the literature to account for the drag reduction effect, and these will now be briefly developed.

Some investigators have asserted that drag reduction results from the viscoelastic effect of polymer solutions on transient shear flows. Ultman and Denn,<sup>3</sup> by using a simple Maxwell model, have shown that flow structures may be changed if the flow velocity exceeds the finite propagation speed of shear waves in a viscoelastic medium. Ruckenstein<sup>4,5</sup> extended this concept to show that the shear stress in an element of liquid in contact with the wall for a given period of time is smaller in the viscoelastic case than in the Newtonian one. His analysis was based on an Einstein and Li renewal model for the wall turbulence<sup>6</sup> and a simplified Oldroyd-Maxwell constitutive equation. Similar results were also obtained independently by Hansen.<sup>7</sup>

The second proposed mechanism for drag reduction emphasizes the high viscosity exhibited by polymer solutions in an elongational flow field. Peterlin<sup>8</sup> indicated that the interaction of each individual polymer molecule with turbulent eddies is important in the sense that polymer molecules locally store energy as a function of the local strain rate of the flow. This change in energy balance produces a significant viscosity contribution as the polymer molecule is exposed to a dilatational flow, the turbulent eddies being effectively damped out. Paterson<sup>9</sup> reasoned that the high solution viscosity in this case is related to the dissipation within polymer coils during deformation in addition to the work required for elastic deformation. Lumley<sup>10</sup> estimated that the elongational viscosity may be as much as four orders of magnitude greater than the shear viscosity. The tendency of such polymer solutions to form filaments under stretching motions is a qualitative example of the effect. Since turbulent drag reduction and high elongational viscosity appear to be the only two large macroscopic effects observed in very dilute polymer solutions, it is possible that the two phenomena are related. Specifically, Kline et al.<sup>11</sup> and Corino and Brodkey<sup>12</sup> reported that the "bursting" process of turbulence near the wall in a turbulent boundary layer is of a stretching nature. Under such circumstances, a high elongational viscosity may be expected to suppress the growth of vortices, thus decreasing the turbulent dissipation and subsequently the wall shear stress.

In this report, these two proposed mechanisms for drag reduction are examined by employing a more complete constitutive equation for polymer solutions. This equation not only enables one to better predict the basic viscoelastic phenomena of polymer solutions from the continuum point of view but also provides a needed correspondence to a popular molecular model. It is hoped that such a continuum approach may shed more light on the merits of each proposed mechanism of drag reduction without losing insight from the molecular level.

### CONSTITUTIVE RELATION

Since the basic mechanisms involved in the drag reduction phenomenon seem to be related to the viscoelasticity of polymer solutions, it was decided

to approach the problem from the continuum point of view by choosing an appropriate constitutive equation relating the stress response of polymer solution to the deformation of the flow. Such a constitutive relation, even if not an ideal one, should at least enable one to predict the three basic rheological functions of a viscoelastic fluid, namely, the shear-dependent viscosity and both the primary and secondary normal stress differences. The importance of the second normal stress difference has been recently pointed out in studies of hydrodynamic stability problems.<sup>13</sup> Furthermore, it is also desirable that the given constitutive equation should ideally correspond to molecular behavioral models if possible. With this goal in mind, it may be useful to briefly discuss the available molecular models as a background for the choice of any constitutive equation.

For the study of nonlinear effects, two molecular models—the simple dumbbell and the more realistic necklace model—are more appropriate than the hydrodynamically equivalent semipermeable sphere.<sup>14</sup> In applying these models, the motion of the free ends held together with elastic forces is considered under hydrodynamic resistance. The necklace model considers the motion of  $N + 1$  beads connected by  $N$  elastic springs. The motion of a molecule can be described by a translation equation and  $N$  total differential equations containing  $N$  eigenvalues  $\lambda_m$  ( $m = 1, 2, 3 \dots N$ ). These equations have exactly the same character as the corresponding equation of the dumbbell model, which yields results corresponding to the first eigenvalue  $\lambda_1$ . Therefore, in those cases where effects mainly depend on the fundamental mode of molecular motion, the difference predicted by the dumbbell model and the necklace model resides primarily in the numerical values of coefficients rather than in any functional dependence. This has been shown in studies applied to intrinsic viscosity and flow birefringence. The dumbbell model, however, is more widely used because of its lesser mathematical complexity. If the motion of the entire polymer molecule is of primary interest very little is lost by this choice of models.

From the continuum point of view, it is now generally accepted that so-called “substructures” exist in systems such as polymer solution. In one specific case,<sup>15</sup> it has been shown that the theoretical results derived from an anisotropic fluid theory of the “structured continuum” type<sup>16</sup> closely describe the experimental behavior of dilute polymer solutions compared with experiments. Recently, Gordon and Schowalter<sup>17</sup> modified Erickson’s anisotropic fluid theory and identified it with the result of the molecular dumbbell theory. They formulated an expression for the time derivative of the end-to-end position vector of a linear macromolecule and combined it with the distribution function for a dilute solution of dumbbell element to obtain an explicit constitutive equation. This equation, based on the combination of continuum and molecular approaches, leads to predictions of both non-Newtonian viscosity and nonzero first and second normal stress differences in simple shearing flow. The second normal stress difference, having the opposite sign of the first one, is negative.

If the stress tensor is defined as

$$\mathbf{T} = -p\mathbf{I} + \boldsymbol{\tau} \quad (1)$$

i.e., sum of the hydrostatic pressure  $p$  and the stress deviator  $\boldsymbol{\tau}$ , then the Gordon-Schowalter model (referred to as G-S model afterward) takes the form

$$\boldsymbol{\tau} = 2\eta_s \mathbf{d} + \boldsymbol{\sigma} \quad (2)$$

where  $\eta_s$  is the solvent viscosity,  $\mathbf{d}$  is the strain rate tensor, and  $\boldsymbol{\sigma}$  obeys a convected Maxwell model,

$$\boldsymbol{\sigma} + \lambda_1 \frac{D\boldsymbol{\sigma}}{Dt} = \frac{2Nc}{M} kT\lambda_1(1 - \epsilon)\mathbf{d} \quad (3)$$

where  $\lambda_1$  is a characteristic stress relaxation time,  $N$  is Avogadro's number,  $M$  is the average molecular weight,  $c$  is the concentration,  $k$  is Boltzmann's constant,  $T$  is the absolute temperature, and  $\epsilon$  is a phenomenological constant subject only to the condition

$$0 \leq \epsilon < 1 \quad (4)$$

and the time derivative is defined as

$$\frac{Db_{ik}}{Dt} = \frac{\partial b_{ik}}{\partial t} + v_j b_{ik,j} + \omega_{ij} b_{jk} + \omega_{kj} b_{ji} - (1 - \epsilon) d_{ij} b_{jk} - (1 - \epsilon) d_{kj} b_{ji} \quad (5)$$

Everage and Gordon<sup>18</sup> obtained an expression for the intrinsic viscosity at zero-shear rate

$$[\eta]_0 = \frac{NkT}{M\eta_s} (1 - \epsilon)\lambda_1 \quad (6)$$

Hence, eq. (3) can be rewritten as

$$\boldsymbol{\sigma} + \lambda_1 \frac{D\boldsymbol{\sigma}}{Dt} = 2c[\eta]_0 \eta_s \mathbf{d} \quad (7)$$

which, together with eqs. (1), (2), (4), and (5) form the complete G-S model.

Since this model is based on a modified molecular theory of dumbbell elements, it is interesting to compare it with another constitutive equation which also has its origin in the dumbbell model. Lumley<sup>19</sup> has recently succeeded in placing the equation relating stress and deformation in a dilute polymer solution through use of the dumbbell model in the form of the Oldroyd constitutive eq. (20),

$$\boldsymbol{\tau} + \lambda_1 \frac{\delta \boldsymbol{\tau}}{\delta t} = 2\eta_0 \left( \mathbf{d} + \lambda_2 \frac{\delta \mathbf{d}}{\delta t} \right) \quad (8)$$

where the time derivative is taken as

$$\frac{\delta b_{ik}}{\delta t} = \frac{\partial b_{ik}}{\partial t} + v_j b_{ik,j} + \omega_{ij} b_{jk} + \omega_{kj} b_{ji} - d_{ij} b_{jk} - d_{kj} b_{ji}, \quad (9)$$

and the constants were interpreted as

$$\lambda_1 = \text{terminal relaxation time} \quad (10)$$

$$\lambda_2 = \lambda_1 / (1 + c[\eta]_0) \quad (11)$$

and

$$\eta_0 = \eta_s (1 + c[\eta]_0). \quad (12)$$

The correspondence between the Oldroyd model and the dumbbell model, as pointed out by Lumley, is essentially independent of flow field. The only assumption is that the molecules are small relative to the smallest scale of the flow. Since under ordinary circumstances the smallest scales of the flow are several orders of magnitude greater than molecular dimensions,<sup>10</sup> this condition is readily met. In Oldroyd's original derivation,<sup>21</sup> the restrictions placed on the material constants assured a positive Weissenberg effect and tension along the streamlines in a cone-and-plate viscometer. These restrictions, in terms of the present notations, are

$$\eta_0 > 0$$

$$\lambda_1 > \lambda_2 \geq 1/8 \lambda_1 > 0$$

or

$$c[\eta_0] \leq 8. \quad (13)$$

For most drag-reducing dilute polymer solutions, Lumley<sup>19</sup> has estimated  $c[\eta_0]$  to be approximately  $10^{-2}$ , which adequately satisfies eq. (13).

It is not surprising that the Oldroyd model has an apparent correspondence to the molecular dumbbell model, since it was originally derived from a structural model for a colloidal suspension in which Hookean elastic spherical particles were supposedly distributed in a Newtonian viscous liquid. Although such a model clearly establishes the applicability of the Oldroyd constitutive equation to the flow of dilute polymer solutions, the constitutive equation fails to predict either a non-Newtonian viscosity or nonvanishing second normal stress difference. It can be shown that the G-S model can be placed in the form of the Oldroyd model with some minor modifications. Substituting eq. (2) into eq. (7), one obtains

$$\tau + \lambda_1 \frac{D\tau}{Dt} = 2\eta_s \left( \mathbf{d} + \lambda_1 \frac{D\mathbf{d}}{Dt} \right) + 2c[\eta]_0 \eta_s \mathbf{d}. \quad (14)$$

Equation (14) may be rewritten by applying eqs. (11) and (12), that is,

$$\tau + \lambda_1 \frac{D\tau}{Dt} = 2\eta_s (1 + c[\eta]_0) \left( \mathbf{d} + \frac{\lambda_1}{1 + c[\eta]_0} \cdot \frac{D\mathbf{d}}{Dt} \right)$$

or

$$\tau + \lambda_1 \frac{D\tau}{Dt} = 2\eta_0 \left( \mathbf{d} + \lambda_2 \frac{D\mathbf{d}}{Dt} \right). \quad (15)$$

This equation is in the same form as eq. (8), except that  $D/Dt$  is now defined as in eq. (5). It is clear that the Oldroyd model is a special case of the G-S model when  $\epsilon = 0$ . It is believed that this modified form of the G-S model can be better applied to the various kinds of dilute polymer solution flows without losing the capability of including all the important basic rheological properties of a viscoelastic liquid. In the following sections, the two mechanisms proposed for explaining drag reduction by dilute polymer solutions will be analyzed with this constitutive equation.

## ANALYSIS

### Case 1: Oscillatory Shearing

Under flow conditions, energy is continuously dissipated as a result of the irreversible work required to deform the fluid against the forces which tend to maintain equilibrium. Consequently, the rate of energy dissipation may be equated to the net power required to sustain the flow and is thus proportional to the viscous shear forces. While the frictional drag is reduced by polymeric additives, the decrease in wall shear stress will reflect a decrease in the rate of energy dissipation. Since the details of turbulent motion are not yet completely well established, it is very difficult to analyze the overall character of steady turbulent flow. However, chaotic turbulent flow can always be Fourier-analyzed as an ensemble of sinusoidal oscillatory motions at discrete frequencies spanning a wide range. Energy spectrum based on frequency can ordinarily be considered as consisting of two parts: (a) an energy-containing range which covers the lower-frequency end representing the motions of large-scale turbulent eddies, and (b) the energy dissipation range which covers the high frequencies corresponding to the motions of small-scale fluctuations. It may therefore be argued that the local energy dissipation properties of a fluid element in a constant frequency sinusoidal oscillatory field may well represent that in a turbulent flow, with respect to a specific frequency component of the fluctuations. Evaluation of the total rate of energy dissipation will involve integrating over the whole frequency spectrum and over the whole domain of the flow region. This last step certainly will depend heavily on the experimental information in the detailed spectrum and turbulence intensity distribution. In this section, the sinusoidal oscillatory motion of a viscoelastic fluid obeying the constitutive eq. (15) will be considered and compared with that in a Newtonian fluid.

The shear behavior of a viscoelastic fluid in an oscillatory motion has been recently analyzed by Darby<sup>22</sup> using a complex viscosity defined as a ratio of oscillatory shear stress  $\tau^*$  and strain rate  $\dot{\gamma}^*$ , i.e.,

$$\eta^*(\omega) = \frac{\tau^*(\omega)}{\dot{\gamma}^*(\omega)} = \frac{\tau_0 e^{i\theta}}{i\omega\gamma_0} = \eta'(\omega) - i\eta''(\omega) \tag{16}$$

where  $\theta$  is the phase angle between oscillatory stress and strain,  $\eta'$  and  $\eta''$  are the in-phase and out-phase components of the complex viscosity, respectively, and

$$\tau^* = \tau_0 e^{i(\omega t + \theta)} = (\tau' + i\tau'') e^{i\omega t} \tag{17}$$

$$\dot{\gamma}^* = \dot{\gamma}_0 e^{i\omega t} = i\omega\gamma_0 e^{i\omega t} = (\dot{\gamma}' + i\dot{\gamma}'') e^{i\omega t}. \tag{18}$$

The local rate of energy dissipation is simply the product of the in-phase stress and strain-rate,

$$P = \text{Re}(\tau^*) \cdot \text{Re}(\dot{\gamma}^*) = -\omega\tau_0\gamma_0 \sin \omega t \cdot \cos (\omega t + \theta). \tag{19}$$

This is averaged over the period of one cycle to give the mean local rate of energy dissipation,

$$\bar{P} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} P dt = 1/2 \omega\tau_0\gamma_0 \sin \theta$$

or

$$\bar{P} = 1/2 \dot{\gamma}'' \tau''. \tag{20}$$

The motion of a fluid element under oscillation can be represented by

$$u_1 = U_1 e^{i\omega t} \tag{21}$$

where the subscript 1 denotes quantities evaluated at the fluid element. This motion will introduce a shear wave propagating in the transverse direction of the oscillation. The fluid in the region of the oscillating element will respond with a velocity  $u^*$ , obeying the equation of motion,

$$\rho \frac{\partial u^*}{\partial t} = \frac{\partial \tau^*}{\partial y}. \tag{22}$$

Substituting eq. (16) and the relations  $u^* = U(y)e^{i\omega t}$  and  $\dot{\gamma} = \partial u^*/\partial y$  into eq. (22),  $U(y)$  can be solved as follows:

$$U(y) = U_1 \exp \left[ -\sqrt{i\omega\rho/\eta^*} \cdot y \right].$$

Hence, the local shear rate on the oscillating element is

$$\dot{\gamma}_1 = \dot{\gamma}^*|_{y=0} = -\sqrt{i\omega\rho/\eta^*} U_1 e^{i\omega t} \tag{23}$$

and the local shear stress is

$$\tau_1 = \tau^*|_{y=0} = \eta^* \dot{\gamma}_1 = -\sqrt{i\omega\rho\eta^*} U_1 e^{i\omega t}. \tag{24}$$

When these results are combined with definitions (17) and (18), the mean local rate of energy dissipation, eq. (20), becomes

$$\bar{P} = \frac{\omega\rho U_1^2}{4} \frac{\eta'}{\sqrt{\eta'^2 + \eta''^2}}. \tag{25}$$

For a viscoelastic fluid obeying the constitutive eq. (15), undergoing an oscillatory motion, the complex viscosity may be calculated in the following manner<sup>23</sup>:

$$\frac{\eta'}{\eta_s} = 1 + \frac{Y}{X} \quad (26)$$

$$\frac{\eta''}{\eta_s} = \frac{Z}{X} \quad (27)$$

where

$$\begin{aligned} X &= (1 + \lambda_1^2 \omega^2)[1 + 4\lambda_1^2 \omega^2 + \epsilon(2 - \epsilon)\lambda_1^2 \dot{\gamma}_0^2] + {}^{3/16} \lambda_1^4 \dot{\gamma}_0^4 \epsilon^2 (2 - \epsilon)^2, \\ Y &= c[\eta]_0 [1 + 4\lambda_1^2 \omega^2 + {}^{1/4} \epsilon(2 - \epsilon)\lambda_1^2 \dot{\gamma}_0^2], \quad \text{and} \\ Z &= c[\eta]_0 \lambda_1 \omega [1 + 4\lambda_1^2 \omega^2 - {}^{1/2} \epsilon(2 - \epsilon)\lambda_1^2 \dot{\gamma}_0^2]. \end{aligned}$$

This result leads to an expression for the mean rate of energy dissipation for the G-S fluid:

$$\bar{P} = \frac{\omega \rho U_1^2}{4} \frac{X + Y}{[X^2 + 2XY + Y^2 + Z^2]^{1/2}} \quad (28)$$

This rate depends on the local shear rate as well as the concentration of polymer solution.

For the special case of  $\epsilon = 0$ , which corresponds to the Oldroyd eq. (8), the complex viscosity becomes

$$\frac{\eta'}{\eta_s} = \frac{1 + \lambda_1 \lambda_2 \omega^2}{\lambda_2 / \lambda_1 + \lambda_2 \lambda_1 \omega^2} \quad (29)$$

$$\frac{\eta''}{\eta_s} = \left( \frac{\lambda_1}{\lambda_2} - 1 \right) \frac{\lambda_1 \omega}{1 + \lambda_1^2 \omega^2} \quad (30)$$

and is identical to that obtained by Williams and Bird.<sup>24</sup> From equations (29) and (30),  $\bar{P}$  thus reduced to

$$\bar{P} = \frac{\omega \rho U_1^2}{4} \frac{1 + \lambda_1 \lambda_2 \omega^2}{\sqrt{1 + \lambda_1^2 \omega^2} \sqrt{1 + \lambda_2^2 \omega^2}}. \quad (31)$$

Since  $\lambda_2 < \lambda_1$ , see eq. (13), it is apparent that viscoelasticity will cause less local power dissipation than would be observed in viscous Newtonian fluids, where the power dissipation is expressed by

$$\bar{P} = \frac{\omega \rho U_1^2}{4}. \quad (32)$$

Since  $\bar{P}$  relates to the power required to sustain flow, this result, as pointed out by Darby,<sup>22</sup> further indicates that less energy is needed to sustain a given fluctuation in a viscoelastic fluid than in a purely viscous fluid of the same viscosity. This leads to the expectation that the viscoelastic effect



would tend to destabilize a flow, i.e., promote transition from a laminar to a turbulent flow. Theoretical analyses<sup>25,26</sup> seem to support this conclusion.

If  $\lambda_2$  is set to be zero in eq. (31), then

$$\bar{P} = \frac{\omega\rho U_1^2}{4} \frac{1}{\sqrt{1 + \lambda_1^2\omega^2}} \tag{33}$$

which is the previous result obtained by Darby<sup>22</sup> and Hansen.<sup>7</sup> For  $\lambda_2 = 0$ , the model reduces to the simple convected Maxwell model, which has been widely applied by many investigators<sup>3,4,7</sup> without any consideration of molecular details or characteristics. However, eq. (11) clearly shows that  $\lambda_2$  can never vanish. Furthermore, if the total stress obeys the Maxwell model, eq. (3), it is equivalent to assume that in eq. (2) the solvent has a zero viscosity, which is certainly unreasonable. These results will be further discussed in later sections.

**Case 2: One-Dimensional Stretching**

Now consider the case of a column of viscoelastic liquid stretched with its axis oriented in the direction of x-axis. The velocity field is represented by

$$\mathbf{v} = [\Gamma(t)x, -1/2\Gamma(t)y, -1/2\Gamma(t)z] \tag{34}$$

which is irrotational in character. The strain rate tensor is considered to be

$$\mathbf{d} = \begin{bmatrix} \Gamma(t) & 0 & 0 \\ 0 & -1/2\Gamma(t) & 0 \\ 0 & 0 & -1/2\Gamma(t) \end{bmatrix}$$

and the condition of incompressibility is satisfied. Only the normal stress components exist, and these can be derived from eq. (15):

$$\begin{aligned} \tau_{xx} + \lambda_1 \left[ \frac{\partial \tau_{xx}}{\partial t} - 2(1 - \epsilon)\Gamma(t)\tau_{xx} \right] \\ = 2\eta_0 \{ \Gamma(t) + \lambda_2[\Gamma'(t) - 2(1 - \epsilon)\Gamma^2(t)] \} \end{aligned} \tag{35}$$

$$\begin{aligned} \tau_{yy} + \lambda_1 \left[ \frac{\partial \tau_{yy}}{\partial t} + (1 - \epsilon)\Gamma(t)\tau_{yy} \right] \\ = \eta_0 \{ \Gamma'(t) + \lambda_2[\Gamma'(t) + (1 - \epsilon)\Gamma^2(t)] \} \end{aligned} \tag{36}$$

and

$$\tau_{zz} = \tau_{yy}. \tag{37}$$

Since these equations are uncoupled in the present situation, they can be readily integrated to give

$$\tau_{zz}(t) = \frac{2\eta_0}{\lambda_1} \left\{ \lambda_2\Gamma(t) + \left( 1 - \frac{\lambda_2}{\lambda_1} \right) \int_0^t e^{-\int_\xi^t \frac{1-2(1-\epsilon)\lambda_1\Gamma(s)}{\lambda_1} ds} \Gamma(\xi) d\xi \right\} \tag{38}$$

$$\tau_{yy}(t) = \frac{-\eta_0}{\lambda_1} \left\{ \lambda_2 \Gamma(t) + \left( 1 - \frac{\lambda_2}{\lambda_1} \right) \int_0^t e^{-\int_0^s \frac{1+(1-\epsilon)\lambda_1\Gamma(s)}{\lambda_1} ds} \Gamma(\xi) d\xi \right\}. \quad (39)$$

The limiting case of constant stretching is most frequently used to interpret physical phenomenon. For  $\Gamma(t) = \Gamma = \text{constant}$ ,

$$\tau_{xx}(t) = 2\eta_0\Gamma \left\{ \frac{\lambda_2}{\lambda_1} + \frac{\left( 1 - \frac{\lambda_2}{\lambda_1} \right)}{1 - 2(1 - \epsilon)\lambda_1\Gamma} \left[ 1 - e^{-\frac{1-2(1-\epsilon)\lambda_1\Gamma}{\lambda_1} t} \right] \right\} \quad (40)$$

$$\tau_{yy}(t) = -\eta_0\Gamma \left\{ \frac{\lambda_2}{\lambda_1} + \frac{\left( 1 - \frac{\lambda_2}{\lambda_1} \right)}{1 + (1 - \epsilon)\lambda_1\Gamma} \left[ 1 - e^{-\frac{1+(1-\epsilon)\lambda_1\Gamma}{\lambda_1} t} \right] \right\}. \quad (41)$$

An elongational viscosity can be defined as

$$\begin{aligned} \bar{\eta}_R = \frac{\tau_{xx} - \tau_{yy}}{\eta_0\Gamma} &= 3 \frac{\lambda_2}{\lambda_1} + \frac{3 \left( 1 - \frac{\lambda_2}{\lambda_1} \right)}{[1 - 2(1 - \epsilon)\lambda_1\Gamma][1 + (1 - \epsilon)\lambda_1\Gamma]} \\ &- 2 \frac{\left( 1 - \frac{\lambda_2}{\lambda_1} \right)}{1 - 2(1 - \epsilon)\lambda_1\Gamma} e^{-\frac{1-2(1-\epsilon)\lambda_1\Gamma}{\lambda_1} t} - \frac{\left( 1 - \frac{\lambda_2}{\lambda_1} \right)}{1 + (1 - \epsilon)\lambda_1\Gamma} e^{-\frac{1+(1-\epsilon)\lambda_1\Gamma}{\lambda_1} t} \end{aligned} \quad (42a)$$

Three remarks concerning this result are in order. First, for the case of a Newtonian fluid,  $\lambda_2/\lambda_1 = 1$ , the elongational viscosity reduces to the classical results of Trouton,<sup>27</sup>  $\bar{\eta}_N = 3$ . Second, the effect of  $\epsilon = 0$  reduces the stretching rate by a factor  $(1 - \epsilon)$  and does not introduce a nonvanishing second normal stress difference in this special case. Third, as time approaches infinity, the elongational viscosity, starting from  $\bar{\eta}_R = 3\lambda_2/\lambda_1$  at  $t = 0$ , reaches the limiting value

$$\bar{\eta}_R = 3 \frac{\lambda_2}{\lambda_1} + \frac{3 \left( 1 - \frac{\lambda_2}{\lambda_1} \right)}{[1 - 2(1 - \epsilon)\lambda_1\Gamma][1 + (1 - \epsilon)\lambda_1\Gamma]} \quad (42b)$$

For  $\epsilon = 0$  and  $\Gamma > 0$  in the stretching case the well-known singularity predicted by the molecular dumb-bell theory,<sup>10,14</sup>  $2\lambda_1\Gamma = 1$ , is exhibited. Based on the present modified dumb-bell model this limiting condition reduces to

$$2(1 - \epsilon)\lambda_1\Gamma = 1. \quad (43)$$

At this stretching rate, eq. (42a) can be expressed as

$$\bar{\eta}_R = \frac{2}{3} + \frac{7}{3} \frac{\lambda_2}{\lambda_1} + 2 \left( 1 - \frac{\lambda_2}{\lambda_1} \right) \frac{t}{\lambda_1} - \frac{2}{3} \left( 1 - \frac{\lambda_2}{\lambda_1} \right) e^{-\frac{3t}{2\lambda_1}}. \quad (44)$$

It can be shown that the result of eq. (42) is equivalent to that of Everage and Gordon<sup>18</sup> through the use of eqs. (11) and (12). This further establishes the validity of the present modification. Equation (15) and the significance of these results will now be developed.

## RESULTS AND DISCUSSION

The local power dissipation of a fluid element in an oscillatory field may be calculated by using eq. (28). Figure 1 shows the result for the simple case  $\epsilon = 0$ , corresponding to that of the Oldroyd fluid. If the dimensionless quantity  $Q = 4\bar{P}/\omega\rho U_1^2$  is plotted against  $\lambda_1\omega$ , it can be seen that the effect of increasing retardation reduces the viscoelastic effect on  $\bar{P}$  at higher frequencies and a minimum is located at values of  $\lambda_1\omega$  of  $O(1)$ . This result immediately points out the shortcoming of the simple convected Maxwell model, where it is assumed  $\lambda_2 = 0$ , in analyzing the transient shear flows. For the case of poly(ethylene oxide), a well-known drag-reducing polymer, at the concentration of 250 ppmw, the ratio of  $\lambda_2/\lambda_1$  is shown in Figure 2 as a function of the average molecular weight. The curve is calculated through use of eq. (11) and the well-known Mark-Houwink-Sakurada relation<sup>28</sup>

$$[\eta]_0 = 1.25 \times 10^{-2} M_w^{0.78} \text{ cc/g.}$$

It should be noted that when the Polyox molecular weight is  $4 \times 10^6$ ,  $\lambda_2/\lambda_1$  assumes a value of about 0.7 as shown in Figure 2. There is essentially no change in  $\bar{P}$  as compared with the Newtonian case where  $Q = 4\bar{P}/\omega\rho U_1^2 = 1.0$ . Clearly, as the concentration decreases further to the usual drag reduction range, say, 25 ppmw,  $\lambda_2/\lambda_1$  will be even closer to 1.0, and the effect on  $\bar{P}$  is further diminished. The effect of  $\epsilon = 0$ , which relates to the non-Newtonian viscosity and nonzero second normal stress in

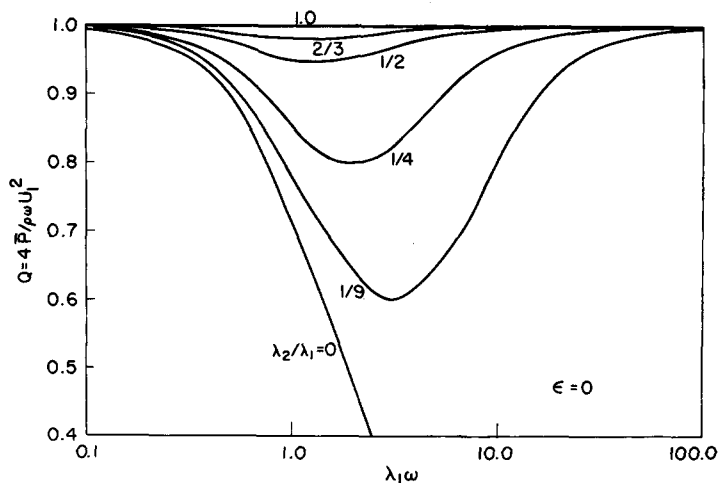


Fig. 1. Effect of retardation on  $\bar{P}$ .

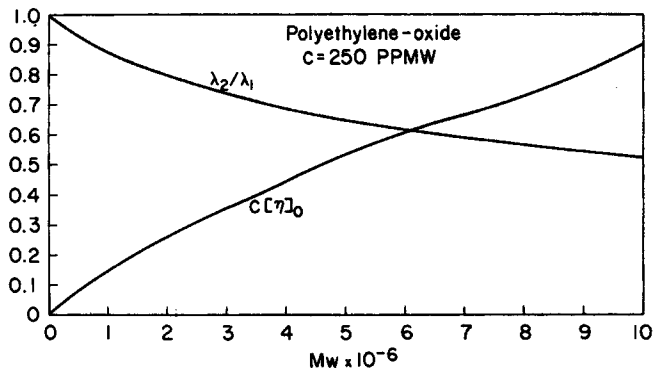


Fig. 2. Retardation time vs. molecular weight for poly(ethylene oxide).

shear flows, is shown in Figure 3 to indicate a negligible effect, since all values of  $Q$  are again very close to 1.0. It is therefore concluded that viscoelasticity, based on the present model, has virtually no effect on the mean local rate of energy dissipation in an oscillatory motion. Consequently, it is felt that the viscoelastic effect on the transient shear flow, as opposed to those conclusions drawn from analyses based on the simple Maxwell model,<sup>4,7</sup> fails to adequately explain the drag reduction effect.

For the case of one-dimensional stretching, results are also presented in Figures 4, 5, and 6. Figure 4 shows that the elongational viscosity approaches a finite asymptote if  $2(1 - \epsilon)\lambda_1\Gamma < 1$ , as predicted by Denn and Marrucci,<sup>29</sup> who used the simple Maxwell model to obtain slightly different results but failed to predict the correct behavior of  $\bar{\eta}_R$  at  $t = 0$ . At the limiting condition of eq. (43), the elongational viscosity increases linearly as time  $t$  becomes large. The limiting slope is proportional to  $1 - (\lambda_2/\lambda_1)$  and decreases as  $\lambda_2/\lambda_1$  increases (see Fig. 5). However, it is interesting to note in Figure 6 that a tremendous increase in  $\bar{\eta}_R$  takes place as soon as  $2(1 - \epsilon)\lambda_1\Gamma$  exceeds 1. For  $(1 - \epsilon)\lambda_1\Gamma = 2.0$ ,  $\bar{\eta}_R$  increases three orders of

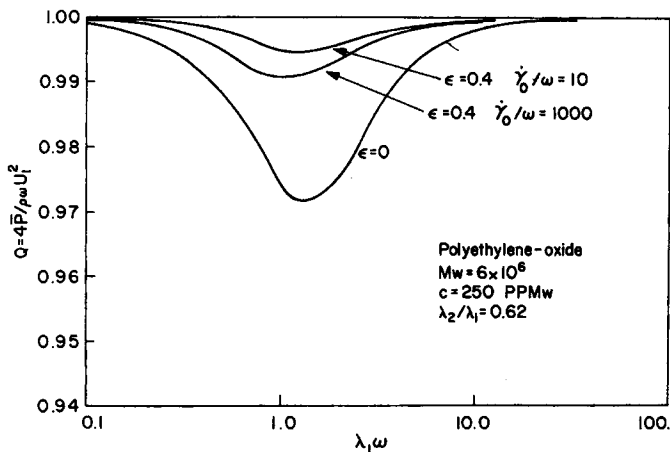


Fig. 3. Effect of  $\epsilon$  and  $\gamma_0$  on  $\bar{P}$ .

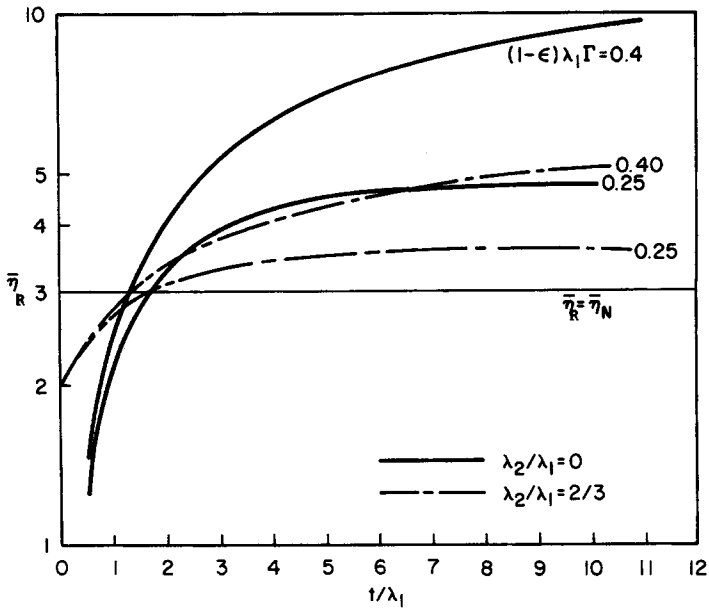


Fig. 4. Elongational viscosity vs. time for  $2(1 - \epsilon)\lambda_1\Gamma < 1$ .

magnitude after a time  $t = 2\lambda_1$ . This phenomenon is better represented in Figure 7 where  $\bar{\eta}_R$  is plotted against  $(1 - \epsilon)\lambda_1\Gamma$ . As time increases, a lower stretching rate is required to reach a very high elongational viscosity. The recent work of Hansen<sup>7</sup> indicated that the onset of turbulent drag reduction is associated with a time scale  $t/\lambda_1 \approx 6.6$ . In that case, it is seen from Figure 7 that large values of  $\bar{\eta}_R$  are easily reached when  $(1 - \epsilon)\lambda_1\Gamma$  slightly exceeds 0.5.

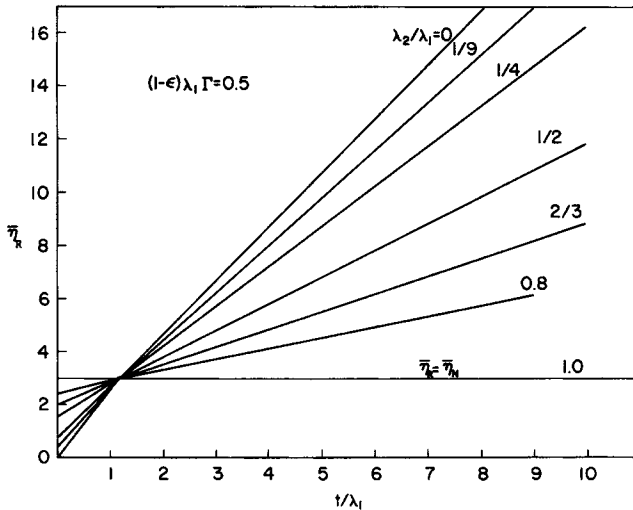


Fig. 5. Elongational viscosity vs. time for  $2(1 - \epsilon)\lambda_1\Gamma = 1$ .

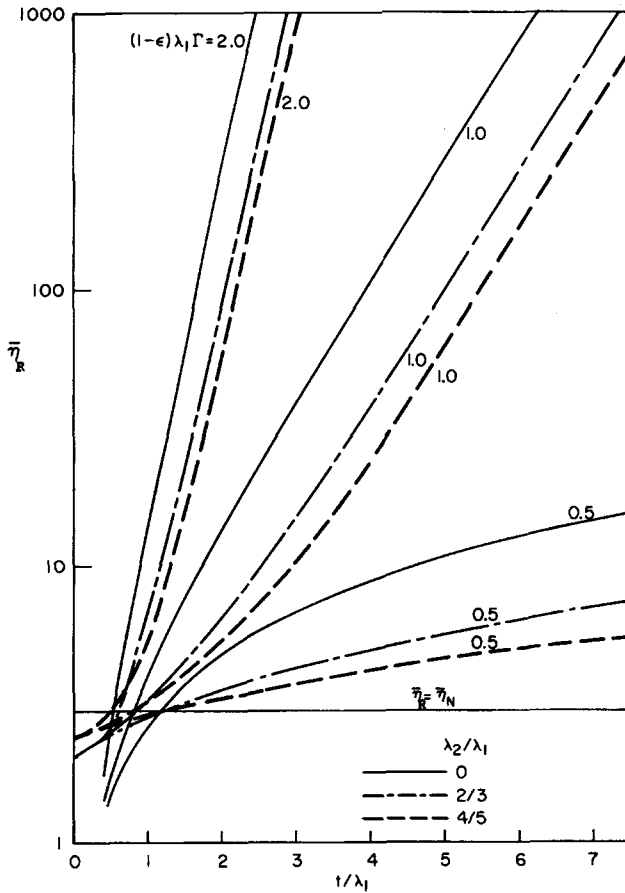


Fig. 6. Elongational viscosity vs. time for  $2(1 - \epsilon)\lambda_1\Gamma > 1$ .

Denn and Marrucci<sup>29</sup> also carried out a simple experiment by stretching a column of a viscoelastic liquid, 1500 ppmw Separan AP-30 in a 15% mixture of glycerin in water. (Separan AP-30 is a partially hydrolyzed polyacrylamide polymer commercially available from the Dow Chemical Company.) By taking high-speed motion pictures, they studied the kinematics during the stretching motion and failed to observe any difference between the Newtonian and the viscoelastic case. This negative experimental finding may be easily explained by noting that the range of  $t/\lambda_1$  in the experiment was only 0.25. Figure 6 indicates that  $\bar{\eta}_R$  is less than  $\bar{\eta}_N$  in Newtonian fluids until  $t \approx \lambda_1$ . The apparent importance of the relative time scale suggests its relevance to the drag reduction effect, a factor also noted by Lumley<sup>10</sup> and others.

In view of the above analysis of the viscoelastic effects on transient shearing and elongational flows, it seems that the latter effect could better explain turbulent drag reduction because of the singular behavior exhibited by eq. (42). As mentioned earlier, extensive studies of the turbulent bursting process near the wall in a turbulent boundary layer<sup>11,12</sup> indicate

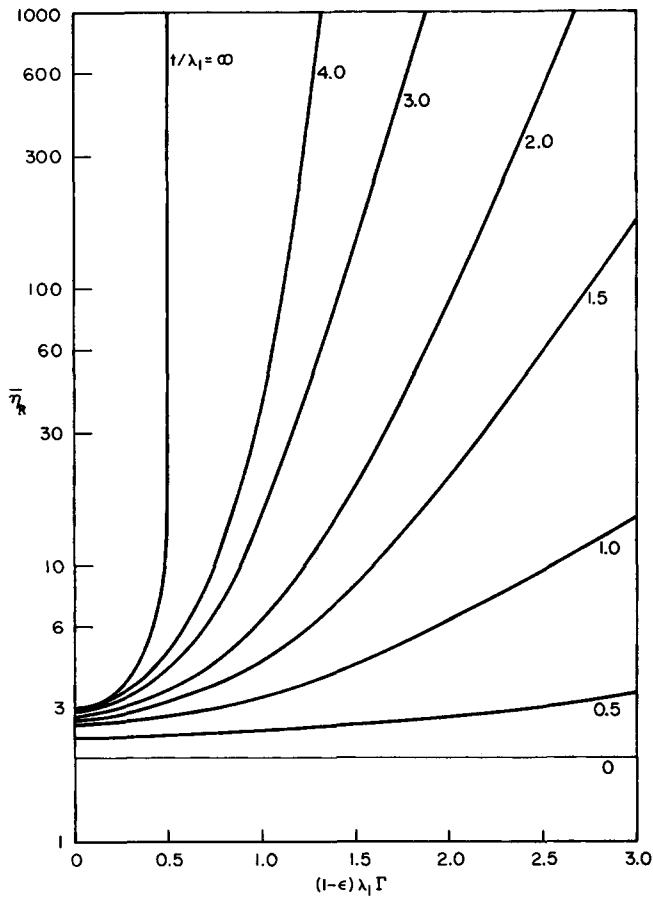


Fig. 7. Elongational viscosity vs.  $(1 - \epsilon)\lambda_1\Gamma$  for  $\lambda_2/\lambda_1 = 2/3$ .

the existence of motions of a stretching nature. When undergoing such elongational deformations, polymer molecules produce extensive viscous dissipation as reflected by the large increase in elongational viscosity. Data reported by Kline et al.<sup>11</sup> show that essentially all turbulence production occurs during the bursting period. The growth of small scale eddies may be suppressed because of elongational viscosity effects on the turbulent bursting process. This reduces the turbulent dissipation in the wall region and therefore causes turbulent drag reduction.

### CONCLUSIONS

By applying a constitutive equation based on a modified molecular dumb-bell model, two proposed mechanisms responsible for drag reduction of dilute polymer solutions in turbulent flow are examined. The viscoelastic effects of polymer solutions on the oscillatory shear flow and the elongational flow are considered separately. Analysis for the case of oscillatory shearing shows that the effect of viscoelasticity on the mean local rate of

energy dissipation of a fluid element oscillating sinusoidally is negligibly small, and indicates the shortcoming of using the simple Maxwell model in predicting the behavior of transient shear flows, especially when the results are applied to explain turbulent drag reduction. On the other hand, the results of analyzing an elongational flow indicate that the elongational viscosity in a viscoelastic fluid may increase to very large values as the stretching rate and the time exceed certain limiting values. It is therefore concluded that turbulent drag reduction may result from the suppression of turbulent vortices during the bursting period because of high elongational viscosity effects involved in the bursting processes in the wall region of the turbulent boundary layer. These results also support the concept that the time scale ratio is the important and relevant factor in the drag reduction effect. More experimental data, however, are needed to estimate the magnitudes of the stretching rates and time scales involved in the turbulent bursting process. With these data, it is hoped that the detailed interaction of polymer molecules with small scale turbulent eddies together with the effect of elongational viscosity on this process may be successfully analyzed.

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Received April 27, 1972

Revised June 14, 1972